Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 97159

B.E./B.Tech. DEGREE EXAMINATION, DECEMBER 2015/JANUARY 2016.

First Semester

Civil Engineering

MA 2111/080030001 — MATHEMATICS – I

(Common to all Branches)

(Regulations 2008)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. The product of two eigen values of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16. Find the third eigen value.

2. When is a Q.F. said to be singular? What is its rank then?

3. Find the equation to the sphere, having the points $(-4, 5, 1)$ and (4, 1, 7) as ends of a diameter.

4. Prove that $9x^2 + 9y^2 - 4z^2 + 12yz - 6zx + 54z - 81 = 0$ represents a cone.

5. Define curvature and radius of curvature.

6. Find the envelope of the family of straight lines given by $y = mx \pm \sqrt{m^2 - 1}$, where m is a parameter.

7. Write the geometrical meaning of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at the point (a, b).

8. If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$, compute $\frac{\partial(u, v, w)}{\partial(x, y, z)}$.

9. Find the value of $\iint xy\, dx\, dy$ taken over the positive quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

10. Find the area of $r^2 = a^2 \cos 2\theta$, by double integration.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Obtain the eigen values and eigen vectors of the matrix $A=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. (8)

(ii) Using Cayley-Hamilton theorem, find the inverse of the matrix $A=\begin{bmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{bmatrix}$ and also verify the theorem. (8)

Or

(b) Reduce $6x^2+3y^2+3z^2-4xy-2yz+4xz$ into a canonical form by an orthogonal reduction. Also find its rank, signature, index and nature. (16)

12. (a) (i) Find the tangent plane to the sphere $x^2+y^2+z^2-4x-2y-6z+5=0$ which are parallel to the plane $x+4y+8z=0$. Find their points of contact. (8)

(ii) Find the equation of the cone whose vertex is at $(1, 1, 3)$ and the guiding curve is $4x^2+z^2=1,\ y=4$. (8)

Or

(b) (i) Find the equation of the sphere passing through the points $(0, 3, 0)$, $(-2, -1, -4)$ and cutting orthogonally the two spheres $x^2+y^2+z^2+x-3z-2=0$ and $2(x^2+y^2+z^2)+x+3y+4=0$. (8)

(ii) Find the equation of the right circular cone generated when the straight line which is the intersection of the planes, $2y+3z=6$ and $x=0$ revolves about the z-axis with constant angle. (8)

13. (a) (i) Find the radius of curvature at any point of the catenary $y = c \cosh \frac{x}{c}$. (8)

(ii) Obtain the equation of the evolute of the parabola $y^2 = 4ax$. (8)

Or

(b) (i) Find the centre of curvature and circle of curvature at $\left(\frac{a}{4}, \frac{a}{4}\right)$ on $\sqrt{x} + \sqrt{y} = \sqrt{a}$. (8)

(ii) Find the envelope of the family of straight lines $\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2$. (8)

14. (a) (i) If $u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$, (8)

(ii) Find the extreme values of $f(x, y) = xy(a - x - y)$. (8)

Or

(b) (i) Expand $e^x \cos y$ in powers of x, y upto the second degree terms using Taylor's theorem. (8)

(ii) Find the greatest and least distances of the point $(3, 4, 12)$ from the unit sphere whose centre is at the origin. (8)

15. (a) (i) Change the order of integration and then evaluate $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy \, dy \, dx$. (8)

(ii) Evaluate $\iiint \sqrt{1 - x^2 - y^2 - z^2} \, dx \, dy \, dz$, taken throughout the volume of the sphere $x^2 + y^2 + z^2 = 1$, by transforming to spherical polar co-ordinates. (8)

Or

(b) (i) Using the transformation, $x + y = u, \ y = uv$, evaluate $\int_0^1 \int_0^{1-x} e^{\frac{y}{x+y}} dy \, dx$. (8)

(ii) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$. (8)